King Chickens[1] Handout #2 Elysée Wilson-Egolf

1. Suppose that pecking were transitive, meaning that given any three chickens, if $C_1 \to C_2$ and $C_2 \to C_3$, then $C_1 \to C_3$. Show that in this case there must be some chicken who pecks all others.

2. Find a flock of six chickens in which every chicken is a king.

3. If a flock has exactly one king, prove that they must peck all the other chickens.

4. “Let K be a king which is pecked by some other chicken. Then there must be a king among the field marshals of K.” Is this statement valid for every pecking order? Either prove this assertion or find a counterexample.

5. Here is an alternate method for defining king chickens. Let us say that to “cull the wimp chickens” means to count the number of other chickens that each chicken pecks, and then remove from the flock the chicken (or the chickens) with the lowest peck count. Cull the flock as many times as possible, then declare all remaining chickens to be kings. Prove that the culling method always produces an odd number of king chickens.

6. It is an unexpected fact that the culling method of finding king chickens might eliminate the chicken(s) with the highest peck count in the original flock. Construct an example of a pecking order in which this occurs.

In the remaining problems, we will say that a flock is equitable if it is possible to arrange all the chickens in a circle so that each one pecks the next going clockwise around the circle.

On the other hand, let us say that the flock has bullies if it is possible to split the flock into two subflocks, each with at least one chicken, so that every chicken in the bully group pecks all the chickens in the other group.

7. Show that if a flock has bullies, then all the kings are bullies.

8. Demonstrate that if a flock is equitable, then it has no bullies.

9. Prove that if a flock has no bullies, then it is equitable. We will accomplish this in two steps. First prove that a ring of some size must exist in which each chicken pecks the next.

10. To finish the previous problem, show that if there are leftover chickens, then it is always possible to incorporate one or more of them to enlarge the ring.

1 These materials taken from Sam Vandervelde’s *Math Circle in a Box*, Chapter 7.