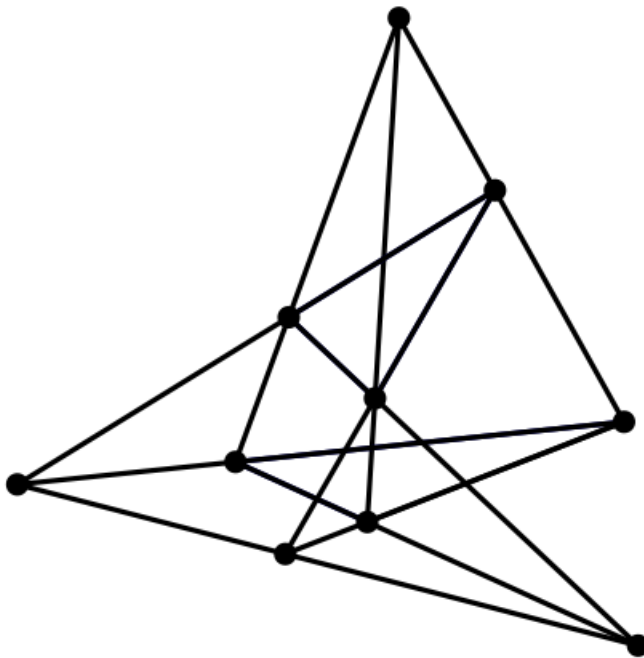# Desargues Configuration, Grids,
# and the Cremona-Richmond Configuration

## Desargues Configuration

1. In this problem, $x, y, z$ are integers.
   a. How many pairs $xy$ are there where $1 \le x < y \le 5$?
   b. How many triples $xyz$ are there where $1 \le x < y < z \le 5$?
   c. Label the points of the Desargues configuration below with pairs $xy$ from part a. and the lines with triples $xyz$ from part a. such that each line $xyz$ has point $xy$, $yz$, and $xz$ on it.
   d. Label the points of the Desargues configuration with triples $xyz$ and the lines with pairs $xy$ such that each point $xyz$ is on lines $xy$, $yz$, and $xz$. (Hint: there is a simple way to do this that comes from your answer to part c.)
   e. Find the number of triangles in the Desargues configuration.
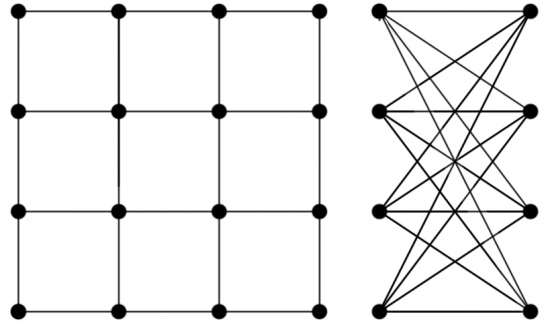


~          Desargues Configuration                     grid of order 3     dual grid of order 3

## Grids and Dual Grids

A *grid* of order $n$ is the points and lines of an $n \times n$ array of $n^2$ squares. A *dual grid* of order $n$ is the points and lines of a graph with two groups of points, each with $n+1$ points, and lines between every pair of points, one from each group. Examples of each are shown above.

2. Find the number of triangles in the grid of order $n$. Find the number of triangles in the dual grid of order $n$.

3. A *quadrangle* is a sequence $p_0, s_1, p_1, s_2, \ldots, s_4, p_4 = p_0$ of four distinct points $p_i$ and four distinct lines $s_i$ where $p_{i-1}$ and $p_i$ are on $s_i$ for $i = 1, 2, 3, 4$ (e.g. a rectangle in the grid). Find the number of quadrangles in the grid of order $n$. Find the number of quadrangles in the dual grid of order $n$.

## Duads, Synthemes, and the Cremona Richmond Configuration

4. A *duad* is a pair $xy$ of integers where $1 \le x < y \le 6$. A *syntheme* is an unordered triple of duads $ab\ cd\ ef$ that includes each of the six digits 1 through 6 exactly once. By unordered we mean that 13 25 46 is the same syntheme as 25 46 13.
    a. How many duads are there?
    b. How many synthemes contain the duad 12?
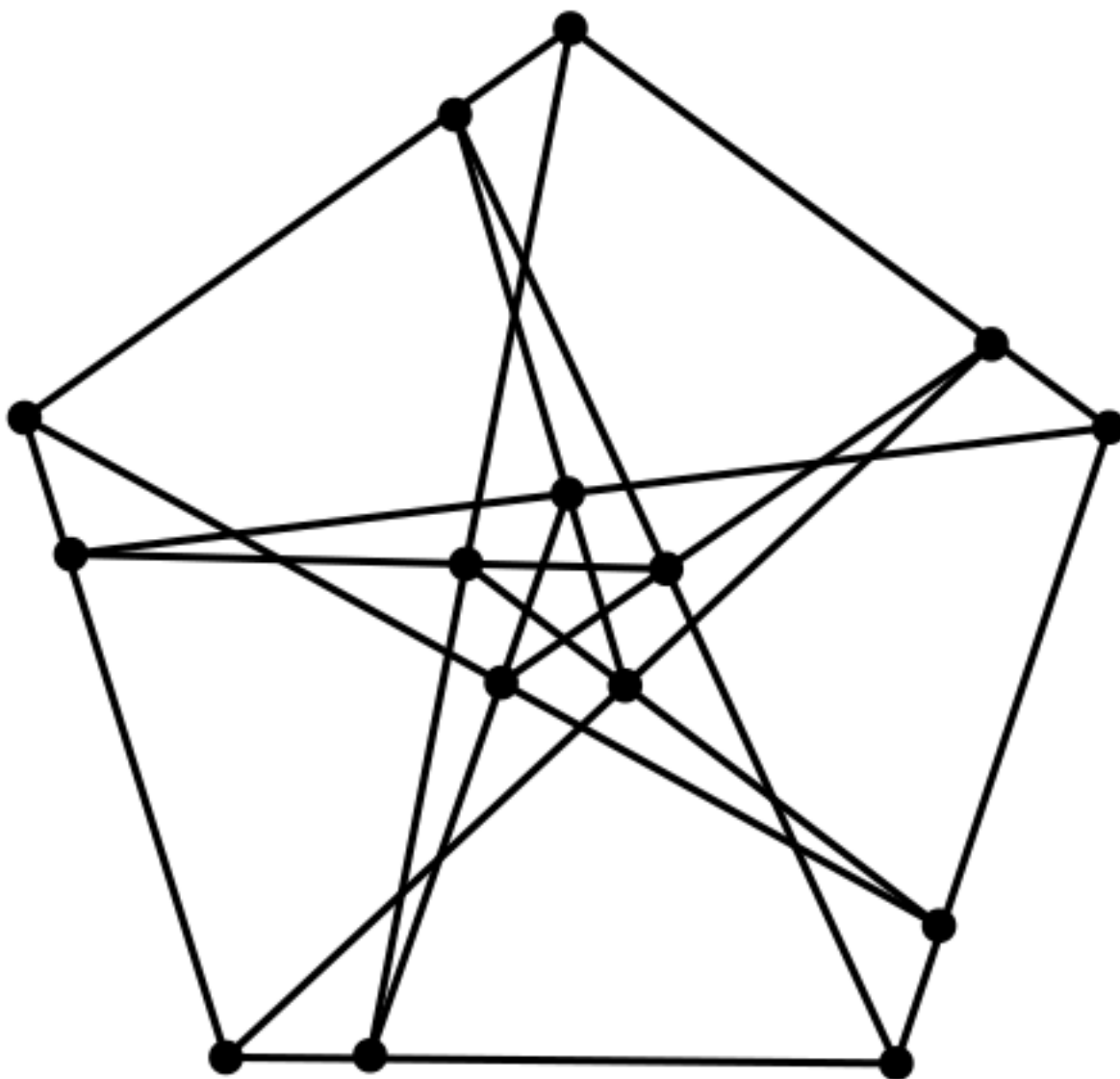    c. How many synthemes are there?

5. The *Cremona-Richmond configuration* (page 3) is a configuration of 15 points and 15 lines such that each point is on three lines and each line is on three points. Label the points of the C-R configuration with the 15 duads so that the three points on each line are labeled with the duads of a syntheme. In this manner the lines are labeled with the 15 synthemes.

6. Another common representation of the Cremona-Richmond configuration (called the "doily") is shown on page 4. In the doily, five lines of the configuration are shown as arcs. Label the points of the doily with the duads so that the points on each "line" are a syntheme.

Problems 7-9 are about the Cremona-Richmond configuration.

7. How many unordered pairs of collinear points are there? How many unordered pairs of non-collinear points?

8. Let $p \neq q$ be collinear points. How many points are collinear with both $p$ and $q$? If $p$ and $q$ are noncollinear, how many points are collinear with both $p$ and $q$?

9. Find the number of quadrangles in the configuration.

# The Cremona-Richmond configuration

# The Cremona-Richmond "Doily"