Logic

Warm-ups

1. Serena Williams beat Martina Hingis in a set of tennis, winning 6 games to 3. There were 5 service breaks, i.e. 5 games were won by the player who did not serve. Who served first?
2. You want to send a valuable object to a friend. You have a box that is more than large enough to contain the object. You have several locks with keys. The box has a locking ring that is more than large enough to have a lock attached. But your friend does not have the key to any lock that you have. How do you do it?
3. My office building authorities announced that a fire drill would be held one day the following week, but the actual day would be a surprise. However, we can prove by induction that the drill cannot be held. Clearly, they cannot wait until Friday, because everyone will know it will be held that day. But if it cannot be held on Friday, then by induction it cannot be held on Thursday, Wednesday, or indeed on any day. What is wrong with this proof?
4. Three playing cards, removed from an ordinary deck of cards, are placed face down in a horizontal row. To the right of a King, there's a Queen or two. To the left of a Queen, there's a Queen or two. To the left of a Heart there's a Spade or two. To the right of a Spade, there's a Spade or two. Name the three cards.

   one Queen or two Queens
5. For every pair of people, assume that they are either friends of each other, or enemies of each other. Prove that any group of 6 people contains either 3 mutual friends or 3 mutual ~~strangers~~ enemies.
6. The famous English mathematician J.E. Littlewood wrote a paper in a French journal. At the end of the paper there were three notes, all in French:
   - I am greatly indebted to Prof. Riesz for translating the present paper.
   - I am greatly indebted to Prof. Riesz for translating the preceding footnote.
   - I am greatly indebted to Prof. Riesz for translating the preceding footnote.

   Assuming that Littlewood was completely ignorant of French, on what grounds did he avoid an infinite regress of identical footnotes by stopping at the third?

Knights and Knaves

A knight always tells the truth. A knave always lies. In the following three problems, everyone is either a knight or a knave.

1. C says, "B is a knave". B says, "A and C are both of the same type – both knights or both knaves." What is A?
2. A says, "B and C are the same type". C is asked, "Are A and B of the same type?" What does C answer?
3. A traveler asks A "Are you a knight?" A, who understands English, but can not speak it, replies, "GOOM!" Then B, who speaks English, explains, "He says Yes." After a pause B adds, "But don't believe him -- he is a knave." What are A and B?

A normal person sometimes tells the truth and sometimes lies. In the following problem, everyone is either a knight, or a knave or normal.

4. A says, “B is a knight.” B says, “A is a knave.” Prove at least one of them is normal.

Ten mutually-referring statements

1. A sheet of paper has statements numbered from 0 to 9. Statement n says, "exactly n of the statements on this sheet are false." Which statements are true and which are false?
2. What if, in the above problem we replace "exactly" by "at least"?
3. Find a number ABCDEFGHIJ such that A is the count of how many 0's are in the number, B is the number of 1's, and so on.

Weighing

1. You are given N balls, and are told that one ball is slightly heavier than the other identical ones. You are given a two-pan balance that lets you put the same number of balls on each side and observe which side (if either) is heavier. What's the minimum number of weighing operations (and way of doing them) that will always find the heavier ball?
2. Same problem as above, but you do not know if the odd ball is slightly heavier or slightly lighter. You have to not only isolate the odd ball, but also determine if it is heavier or lighter.
3. You have ten boxes; each contains nine balls. The balls in one box weigh 0.9 kg; the rest weigh 1.0 kg. You have scale that gives a digital readout. Using one weighing on this scale, find the box containing the light balls.

Gradual Information

1. There are three men, A, B, C, sitting one behind another. A can see B and C; B can see C; and C can see no one. An umpire has 3 red hats and 2 black hats. The umpire tells them to close their eyes, and puts a red hat on everyone’s head. Then he asks A, “Do you know the color of your hat?” A says no. He asks B and B also says no. He asks C, and C says, “Yes, my hat is red.” How did C know?
2. Generalize to n people, n red hats and (n-1) black hats.
3. Two positive integers, a and b, not necessarily distinct, are chosen such that a, b > 1 and a + b <= 100. Mathematician S is told the sum a + b (only), mathematician P is told the product a * b (only). P says to S: "I don't know your sum." S says to P: "Yes I knew that." Later P says to S: "Now I know your sum." Then S says to P: "Now I know your product." What are the values of a and b?
4. Two logicians place cards on their foreheads so that what is written on the card is visible only to the other logician. Consecutive positive integers have been written on the cards. The following conversation ensues:
   - A: "I don't know my number."
   - B: "I don't know my number."

- A: "I don't know my number."
- B: "I don't know my number."
- ... n statements of ignorance later ...
- A or B: "I know my number."

What is on the card and how does the logician know it?

Who's who

1. Three young couples went to a dance. One girl was dressed in red, one in green, one in blue. The boys were in outfits of the same three colors. When all three couples were dancing, the girl in green said to her partner, the boy in blue: "Isn't it funny that no one is dancing with a partner dressed in the same color?" What is the color of the partner of the girl in red?
2. Paul, John, George are three rock stars. One plays a guitar, one plays the drums, one plays the piano, but not necessarily in that order.
   - The drummer tried to hire the guitarist for a recording session, but was told that he was out of town, doing shows with the pianist.
   - The drummer admires the work of both musicians
   - The pianist earns more money than the drummer
   - Paul earns less than John
   - George has never heard of John

   What instrument does each person play?
3. Four college girls who share an apartment are listening to an album of music, while one does her nails, one does her hair, one puts on make-up and one is reading.
   - Myra isn't doing her nails and she isn't reading
   - Maud is not putting on make-up and she is not doing her nails
   - Mary is not reading and she is not doing her nails
   - Mona is not reading and she is not putting on make-up
   - If Myra is not putting on make-up, then Mona is not doing her nails

   Who's doing what?
4. Professor Merle White, Leslie Black and Jean Brown were lunching together in the University cafeteria.
   - "Isn't it remarkable," observed the lady, "that our last names are Black, White and Brown, and one of us has black hair, one brown hair and one white."
   - "It is indeed," replied the person with black hair, "and have you noticed that not one of us has hair that matches his or her name?"
   - "By golly, you're right!" exclaimed Professor White.

   If the lady's hair isn't brown, what is the color of Professor Black's hair?
5. On a train, Smith, Robinson and Jones are the fireman, brakeman, and engineer, but not necessarily respectively. Also aboard the train are three businessmen who have the same names. To distinguish them from the railway employees, let us prefix a "Mr." to their names: Mr. Smith, Mr. Robinson, and Mr. Jones.
   - Mr. Robinson lives in Detroit
   - The brakeman lives exactly half way between Chicago and Detroit.
   - Mr. Jones earns exactly $20,000 per year.

- The brakeman's nearest neighbor, one of the passengers, earns exactly three times as much as the brakeman.
- Smith beats the fireman at billiards.
- The passenger whose name is the same as the brakeman's lives in Chicago.

Who is the engineer?

6. (invented by Raymond Smullyan)
   - In 1918, on the day that armistice of World War 1 was signed, three married couples celebrated by having dinner together.
   - Each husband is the brother of one of the wives, and each wife is the sister of one of the husbands; that is there are three brother-sister pairs in the group.
   - Helen is exactly 26 weeks older than her husband, who was born in August.
   - Mr. White's sister is married to Helen's brother's brother-in-law. She (Mr. White's sister) married him on her birthday, which is in January.
   - Marguerite White is not as tall as William Black.
   - Arthur's sister is prettier than Beatrice is.
   - John is fifty years old.

   What is Mrs. Brown's first name?