

Monthly Contest 2
Due December 8, 2017

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any book or website (including forums) unless otherwise stated in the problem.**

Problems

1. The increasing sequence of positive integers a_1, a_2, a_3, \dots has the property that $a_{n+2} = a_n + a_{n+1}$, for all $n \geq 1$. Suppose that $a_7 = 120$. What is a_8 ?

2. Let $0 < \theta < 45$. Arrange

(a) $(\tan \theta)^{\tan \theta}$ (b) $(\tan \theta)^{\cot \theta}$
(c) $(\cot \theta)^{\tan \theta}$ (d) $(\cot \theta)^{\cot \theta}$

in decreasing order.

3. A heap of balls consists of one thousand 10-gram balls and one-thousand 9.9 gram balls. we wish to pick out two heaps of balls with equal numbers of balls in them but different total weights. What is the minimum

number of weighings required to do this? (The balance scale reports the weight of the objects in the left pan minus the weight of the objects in the right pan.)

4. Prove that the number of partitions of any positive integer n into distinct parts is equal to the number of partitions of n into odd parts. (Example: The partitions of 6 into distinct parts are 6, 5+1, 4+2, 3+2+1. The partitions of 6 into odd parts are 5+1, 3+3, 3+1+1+1, and 1+1+1+1+1+1.)
5. Determine, with proof, if it is possible to arrange 1, 2, ..., 1000 in a row such that the average of any pair of distinct numbers is not located in between the two numbers.