Monthly Contest 2
Due December 8, 2017

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any book or website (including forums) unless otherwise stated in the problem.**

Problems

1. How many ways can a \$100 bill be changed into \$1, \$2, \$5, \$10, \$20, and \$50 bills?

2. The increasing sequence of positive integers $a_1, a_2, a_3, ...$ has the property that $a_{n+2} = a_n + a_{n+1}$, for all $n \leq 1$. Suppose that $a_7 = 120$. What is a_8?

3. Let $0 < \theta < 45$. Arrange

   (a) $(\tan\theta)^{\tan\theta}$ (b) $(\tan\theta)^{\cot\theta}$

   (c) $(\cot\theta)^{\tan\theta}$ (d) $(\cot\theta)^{\cot\theta}$

   in decreasing order.

4. What is the largest positive integer n for which $n^3 + 100$ is divisible by $n + 10$?

5. Let S be a set of n persons such that

   (a) Any person is acquainted to exactly k other persons in S

   (b) Any two persons that are acquainted have exactly l common acquaintances

   (c) Any two persons that are not acquainted have exactly m common acquaintances in S.

   Prove that $m(n-k) - k(k-l) + k - m = 0$.