Monthly Contest 2
Due December 14, 2016

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

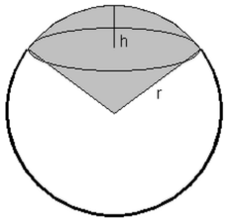
DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. You may view any book or website (including forums) as long as you do not communicate the problem in any sort of manner (e.g. posting the problem statement or variations on a forum).

Problems

1. Given a regular convex pentagon ABCDE and 6 colors,
   (a) Find the number of all possible colorings of the vertices such that no edges connect vertices of the same color.
   (b) Find the number of all possible colorings of the vertices such that no diagonals connect vertices of the same color (edges can connect vertices of the same color).
2. Find the remainder when $e^{2016(ln7)+7(ln2016)}$ is divided by 11.
3. Find the volume of the solid which is the intersection of two unit spheres with their centers one unit apart. (Hint: The area of a spherical sector

is $\dfrac{2\pi r^2 h}{3}$, where r and h are as labeled in the figure below.)



4. Find all natural numbers a, b, c such that a!+b!=c!. Prove that you have found all of them.

5. Nina chooses a coin from one hundred pennies and promptly tosses six heads in a row. However, she soon discovers that four of the 100 pennies were two-headed. Find the probability that Nina chose one of the two-headed pennies given that she flipped six heads in a row.