Russian Math Circle Problems

Fall 2010

Instructions: Work as many problems as you can. Even if you can't solve a problem, try to learn as much as you can about it. Please write a complete solution to each problem you solve, as if you were entering it into a math contest and had no ability to explain it to the grader. This will help you make sure that you've thought of all the possibilities.

1. In the expression below, insert parentheses and/or signs $+$, $-$, $\times$, $/$ in order to get a true equation:
$$1/2 \quad 1/6 \quad 1/6030 = 2010$$

2. Solve the cryptogram: $SEND + MORE = MONEY$. Each letter represents a digit.

3. Find every integer which is equal twice the sum of its digits.

4. Paint every point of the plane using exactly three colors so that every line contains points of exactly two colors. How many ways can you think of to do this?

5. The numbers from 1 to 64, inclusive, are placed in an 8×8 grid, one number per square. If the sum of each row is the same, we call the arrangement BALANCED. If the sum of the rows are all different, we call the arrangement SILLY. Prove that the number of balanced arrangements is less than or equal to the number of silly arrangements.

6. In trapezoid $ABCD$ with BC parallel to AD, let $BC = 1000$ and $AD = 2008$. Let the angle A be $37°$, angle D be $53°$, and M and N be the midpoints of BC and AD, respectively. Find the length MN. (AIME, 2008, #5).

7. Find $ax^5 + by^5$ if the real numbers a, b, x and y satisfy:
$$\begin{aligned} ax + by &= 3 \\ ax^2 + by^2 &= 7 \\ ax^3 + by^3 &= 16 \\ ax^4 + by^4 &= 42 \end{aligned}$$
(AIME, 1990, #15)