

# Russian Math Circle Homework Problems

**Instructions:** Work as many problems as you can. Even if you can't solve a problem, try to learn as much as you can about it.

1. Find a number that uses each of the digits 1 through 9 exactly once, such that the number formed by looking at the first  $n$  digits is divisible by  $n$ ,  $n=2, 3, \dots, 9$ .
2. Prove that if  $p$  and  $p^2 + 2$  are prime, then so is  $p^3 + 2$ .
3. A laser beam is shot from the southwest corner of a square made out of a reflective material. The beam is aimed at the northeast corner; if it exactly hits this corner or any other corner, it is absorbed. If it hits any other place, it bounces. It turns out that the aim was not quite perfect, and the beam bounced 2009 before it got absorbed.
  - (a) Assuming that the aim was as good as possible (i.e., as close to 45 degrees as can be), what was the angle?
  - (b) How many angles would yield 2009 bounces?
  - (c) What if it was a cube instead of a square?
4.
  - (a) Is there a set of points in the plane which meets every circle in exactly two points?
  - (b) Is there a set of points in the plane which meets every circle of radius 1 in exactly two points?
5. Which of the two numbers is bigger:  $9^{9^{\dots}}$  (one hundred exponents) or  $9!!!\dots!$  (one hundred factorials)?