Optimal Stopping Rules

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October 28, 2009
San Jose Math Circle

A.

1. You get to roll a fair die up to n times in succession. After each roll you can decide whether to stop with the value of that roll or continue. Whatever value you stop with is the amount you win. Assuming that your goal is to maximize your expected winnings, what rule should you use to decide whether to stop or not after the first roll of the die when $n = 2$? When $n = 3$? When $n = 4$? When $n = 5$? When $n = 6$?

2. Find the amount you can expect to win if you simply get the largest value rolled in n rolls of the die for $n = 2, 3, 4, 5, 6$.

3. Modify problem 1 so that instead of rolling a die, you get a random number between 0 and 1 from your calculator. Again you want to maximize the value you stop with when you are allowed up to n random numbers. For $n = 2$ and $n = 3$ find the stopping rule you should use.

4. Continuing with problem 3, let w_n be the average amount you win. You can express w_n in terms of w_{n-1}, so that you can recursively compute w_n starting with $w_1 = 1/2$. What is the formula? Make sure you check it for w_2 and w_3 using your answers to the previous problem.

5. What do you expect to happen to w_n as n gets larger and larger? Can you draw a plot to explain that?

6. (Requires calculus) What is the average largest value among n random numbers drawn from the interval $[0, 1]$?