Problem Challenge by Tom Rike for the SJMC talk "Heptadecagon" on October 21, 2009

1. Fermat thought that the converse of his theorem " If p is a prime number then $a^p \equiv a \bmod p$ " is also true. Show that it is false by showing the $2^{341} \equiv 2 \bmod 341$ and that 341 is not prime.

2. Show that $F_5 = 2^{2^5} + 1$ is not prime by finding one of the factors using only five carefully chosen divisions.

3. Consider a pentagon, hexagon and decagon inscribed in a circle. Prove that the square of a side of the pentagon is equal to the sum of the squares of a side of the hexagon and a side of the decagon.

4. Can you express the side of a 17-gon inscribed in a unit circle using only addition, subtraction, multiplication, division, and square roots?