# Infinity Problems

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One of the most important ideas involved when thinking about finite and infinite sets is the concept of cardinality. Two sets have the same cardinality (which basically means they have the same number of elements, or are the same size) if you can match up all the elements in one with all the elements in the other in such a way that every element in one set is matched with exactly one element in the other set and none are left out. A mathematician would say that there exists a 1-1 onto mapping from one set to the other.

For example, if we want to show that the sets $\{1, 2, 3, 4\}$ and $\{A, B, C, D\}$ have the same cardinality (are the same size), we can do that by exhibiting the following match:

$$1 \leftrightarrow A, 2 \leftrightarrow B, 3 \leftrightarrow C, 4 \leftrightarrow D$$

Of course many other matchings are possible and any one would show that the two sets are the same size.

The following problems relate to this idea of matching sets. They are listed in approximately increasing order of difficulty. The last one is quite difficult:

1. How many different ways are there to match up the two sets $\{1, 2, 3, 4\}$ and $\{A, B, C, D\}$?

2. Show that the set of natural numbers ($\{0, 1, 2, 3, \ldots\}$) can be matched with the set of natural numbers that are multiples of 7 ($\{0, 7, 14, 21, 28, \ldots\}$).

3. Show that the set of natural numbers ($\{0, 1, 2, 3, 4, \ldots\}$) can be matched with the set of integers ($\{\ldots, -2, -1, 0, 1, 2, \ldots\}$).

4. Can you match up the real numbers in $[0, \infty) = \{x : x \geq 0\}$ with the real numbers in $(0, \infty) = \{x : x > 0\}$?

5. Show that you can match the set of all polynomials in $x$ having integer coefficients with the set of natural numbers.