

Russian Math Circle Homework Problems

April 9, 2009

Instructions: Work as many problems as you can. Even if you can't solve a problem, try to learn as much as you can about it.

1. I have five cards with four numbers on each one of them. All the numbers from 1 to 20 are on them. The sums of all 4 numbers on all the cards are the same. Card 1 has 17 and 2. Card 2 has 6 and 11. Card 3 has 15 and 3. Card 4 has 10 and 18. Card 5 has 4 and 19. What are the other two numbers on each one of the cards?

2. If we write the numbers in normal base-10 form from 1 to N next to each other to make a larger number, can that number be a palindrome for any $N > 1$?

NOTES: (i) A *palindromic* number is one that reads the same left-to-right as right-to-left, for example: 1, 343, 1234321.

(ii) If $N = 12$, the number we'd obtain is: 123456789101112.

3. Find all 5-digit numbers which, when divided by 4, yield a 5-digit number that is the same as the original, but with the digits reversed.

4. Three circles with radii R_1 , R_2 , and R_3 are pairwise tangent. Let P be the point of tangency of the first two circles, and ℓ be their common tangent passing through P . Find the length of the segment of ℓ that lies inside the third circle.