Russian-Style Math Circle Problems

All the following problems are taken from the individual round questions from ARML contests. They are of varying difficulty, and some are much easier than they look. You are not expected to work all of them in a single circle session; choose a couple of them that are interesting to you and work on them.

1. ARML 1999: A pro athlete played for 17 years and earned 72 million dollars. She was paid k million dollars each year where k is an integer and received an extra one million dollars each year her team made the playoffs. Compute the number of years her team made the playoffs.

2. ARML 1995: Find the largest prime factor of:

$$3(3(3(3(3(3(3(3(3(3+1)+1)+1)+1)+1)+1)+1)+1)+1)+1.$$

3. ARML 1995: Compute the number of distinct planes passing through at least three vertices of a given cube.

4. ARML 1996: The roots of $ax^2 + bx + c = 0$ are irrational, but they are approximately 0.8430703308 and -0.5930703308. If a, b and c are integers, and $a > 0$, $|b| < 10$ and $|c| < 10$, find a, b and c.

5. ARML 1997: Let a, b, c and n be positive integers. If $a + b + c = (19)(97)$ and $a + n = b - n = c/n$, compute the value of a.

6. ARML 1998: Compute:

$$199919981997^2 - 2 \cdot 199919981994^2 + 199919981991^2.$$

7. ARML 1999: Arrange the following products in increasing order from left to right:

$$1000! \quad (400!)(400!)(200!) \quad (500!)(500!) \quad (600!)(300!)(100!) \quad (700!)(300!).$$

8. ARML 2000: If, from left to right, the last seven digits of $n!$ are 8000000, compute the value of n.

9. ARML 2001: The vertices of a regular dodecagon (12-sided figure) are given by (x_i, y_i), for $i = 1, 2, 3, \ldots, 12$. if $(x_1, y_1) = (15, 9)$ and $(x_7, y_7) = (15, 5)$, compute:

$$\sum_{i=1}^{12}(x_i - y_i) = (x_1 - y_1) + (x_2 - y_2) + \cdots + (x_{12} - y_{12}).$$

10. ARML 2003: Compute the largest factor of 1001001001 that is less than 10000.