Russian Math Circle Problems

December 6, 2007

1. Look at the final (terminal) digit of the numbers: $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ and 2^8. What is the final digit of 2^{20}? of 2^{125}? of $2^{1234567}$?
2. Find the final *two* digits of $7^1, 7^2, 7^3, 7^4, 7^5, 7^{41}$ and 7^{12345}.
3. Suppose we calculate exactly the value of $(732.15)^{431}$. What are the final three digits in the decimal expansion, before it terminates with zeroes?
4. How many terminal zeroes are there if you expand $1000!$ completely. $(1000! = 1000 \times 999 \times \cdots 2 \times 1)$.
5. Provide a complete factorization of $23!$ into primes.
6. What is the remainder when 3^{2007} is divided by 7?
7. Find all numbers n that satisfy the following condition: among the numbers $1, 2, 3, \ldots, 999, 1000$, there are exactly 10 of them such that the sum of the number's digits is exactly n.
8. Does there exist a triangle such that the sum of two of its altitudes is larger than the sum of the two corresponding bases?
9. Four people move along a road: one is driving a car, the second is riding a motorcycle, the third is riding a moped, and the fourth, a bicycle. Each maintains a constant speed. The car driver met the moped at 12:00 pm, met the bicyclist at 2:00 pm, and met the motorcyclist at 4:00 pm. The motorcyclist met the moped at 5:00 pm, and met the bicyclist at 6:00 pm. At what time did bicyclist meet the moped? (In this problem, the word "met" simply means that the vehicles are in the same place; they could be going in the same or opposite directions.)
10. Is it possible to place numbers $1, 2, 3, 4, 5, 6, 7, 8$ at the vertices of a regular octagon in such a way that the sum of every three neighboring numbers is: (a) larger than 11; (b) larger than 13?