# Committees and Configurations

## Committees

A corporation is governed by a board of directors. Often a corporation forms committees comprised of directors from the board to oversee its business operations. In the following problems we will assume that:

(1) Each director belongs to the same number of committees.

(2) Each committee has the same number of members.

(3) Two directors are on at most one committee together.

We will use the following variables:

$$\begin{aligned} v &= \text{number of directors,} \\ r &= \text{number of committees each director belongs to,} \\ b &= \text{number of committees,} \\ k &= \text{number of directors on each committee.} \end{aligned}$$

In answering the problems, it is convenient to use letters $a, b, c, \ldots$ to represent the directors and use a string of letters, such as $bce$, to represent a committee.

1. There are six directors on the board of Hexacomm Data Corporation.
   a. The directors would like committees to have two members each. If each director is on three committees ($v = 6, r = 3, k = 2$), how many committees would there be (i.e. what is $b$)? Show how the committees may be formed, or prove that it is not possible.
   b. The chairman of the board decides that there are too many committees so he wants to reduce the number of committees to six but increase the size of each committee to three ($v = 6, r = 3, b = 6, k = 3$). Show how the committees may be formed, or prove that it is not possible.

2. The board of Heptad Inc. has seven directors.
   a. The directors would like to form seven committees each with three members ($v = 7, b = 7, k = 3$). Assuming this is possible, how many committees must each director belong to (i.e. what is $r$)? Show how the committees may be formed, or prove that it is not possible.
   b. After a few years, the shareholders demand another director. Consequently Heptad Inc. renames itself Octad Inc. and increases the size of the board to eight. Octad keeps the committee size at three and continues to have each director belong to three committees ($v = 8, r = 3, k = 3$). Assuming this is possible, how many committees must they have (i.e. what is $b$)? Show how the committees may be formed, or prove that it is not possible.

3. There are 10 directors on the board of Starlight Corporation.
   a. The directors would like to form five committees. If each director is on two committees ($v = 10, r = 2, b = 5$), then how many directors must belong to each committee (i.e. what is $k$)? Show how the committees may be formed, or prove that it is not possible.
   b. After a few years, the company realizes that it needs more committees and wants to increase the number of committees to 10, while reducing the size of each committee to three. ($v = 10, b = 10, k = 3$). Assuming this is possible, how many committees must each director belong to (i.e. what is $r$)? Show how the committees may be formed, or prove that it is not possible.

## Configurations

Problems 4-6 refer to the geometric configurations $9_3$ A, $9_3$ B, and $9_3$ C on page 4. The three configurations are realizations of the three distinct $9_3$ combinatorial configurations.

4. Determine which configuration is the Pappus configuration.

5. Configurations 1 and 2 below are the configuration tables for the two $9_3$ non-Pappus configurations. Match them to the geometric configurations in the figures.

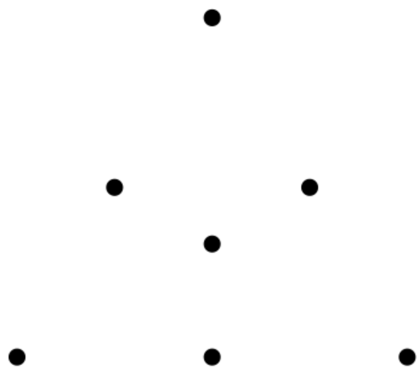
| Configuration 1 | Configuration 2 | Cyclic $9_3$ Configuration |
|---|---|---|
| $a\ b\ c$ | $a\ b\ c$ | *0 1 3* |
| $a\ d\ e$ | $a\ d\ e$ | *1 2 4* |
| $a\ f\ g$ | $a\ f\ g$ | *2 3 5* |
| $b\ d\ f$ | $b\ e\ h$ | *3 4 6* |
| $b\ g\ h$ | $b\ f\ i$ | *4 5 7* |
| $c\ e\ h$ | $c\ d\ i$ | *5 6 8* |
| $c\ g\ i$ | $c\ e\ g$ | *6 7 0* |
| $d\ h\ i$ | $d\ f\ h$ | *7 8 1* |
| $e\ f\ i$ | $g\ h\ i$ | *8 0 2* |

6. Which configuration is a realization of the cyclic $9_3$ configuration table above?

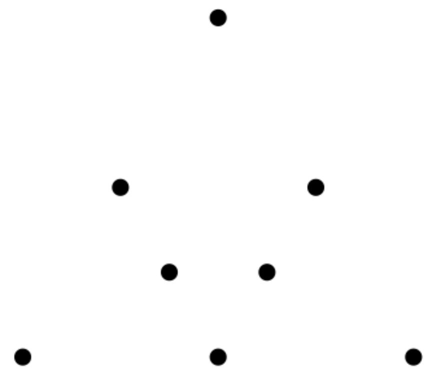
Problems 7-8 refer to the geometric configurations $10_3$ A, $10_3$ B, and $10_3$ C on page 4.

7. Write the configuration table of the cyclic $10_3$ configuration generated by the line *0,1,3*. Determine which of the three configurations A, B, or C is the cyclic configuration and label the points according to your table.

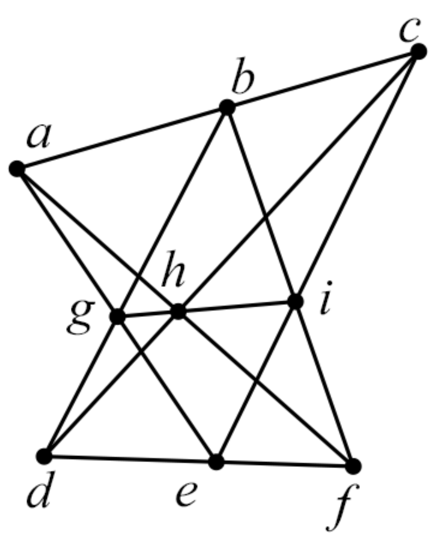
8. Prove that none of the three configurations A, B, C is the Desargues configuration.



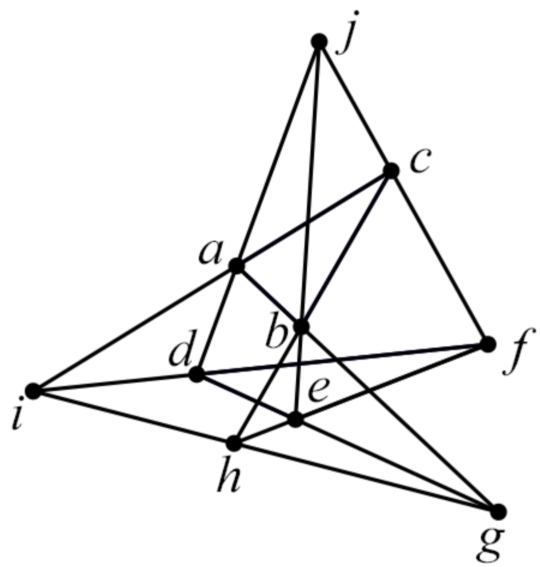
Fano configuration points



Möbius-Kantor configuration points

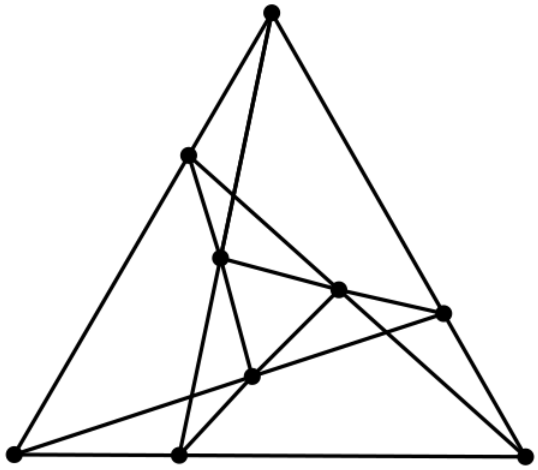



Pappus configuration

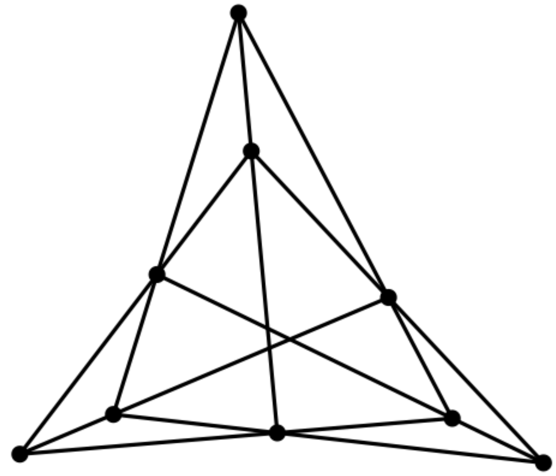



Desargues configuration

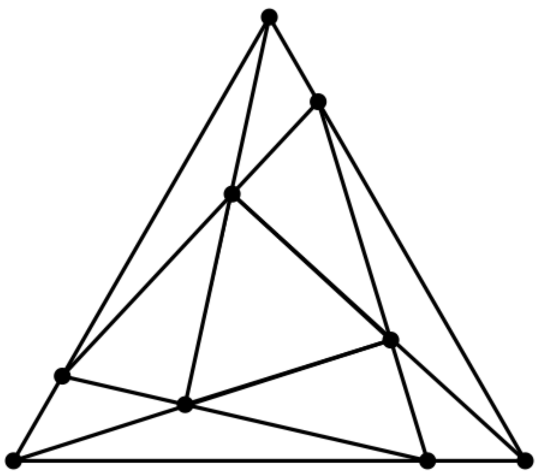
Draw your own Pappus and Desargues configurations here



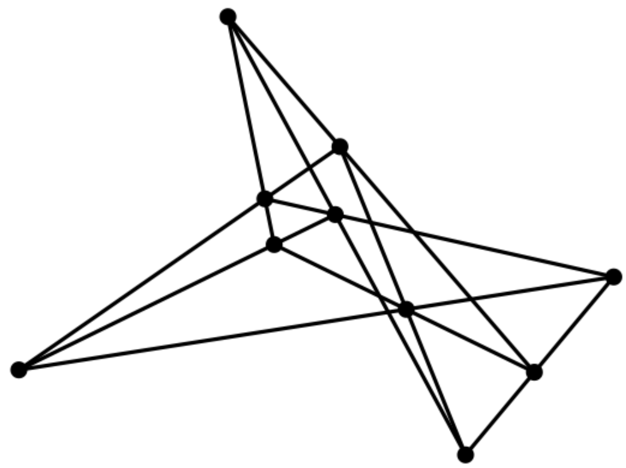
$9_3$ A



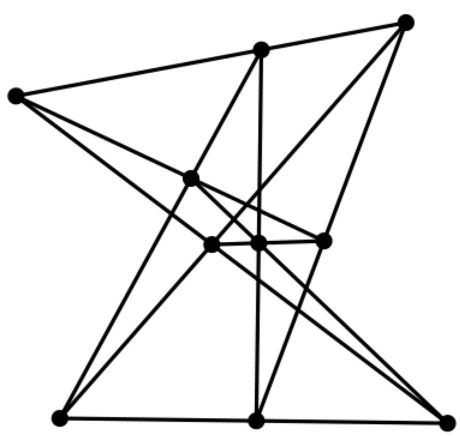
$9_3$ B



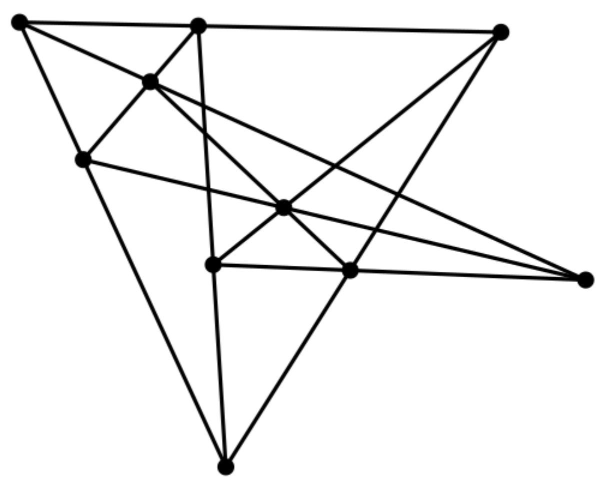
$9_3$ C



$10_3$ A



$10_3$ B



$10_3$ C