Monthly Contest 2
Due February 1, 2019

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any website (including forums) unless otherwise stated in the problem.**

Problems

1. My friend goes to a cafe every morning, arriving at times evenly distributed from 5:00 to 6:00. I go to the same cafe, but from 5:15 to 6:15. My friend spends n minutes there, I spend 15. If the expected number of times we meet at the cafe in a week is 3, find n.

2. Evaluate:

$$2\binom{10}{0}+3\binom{10}{1}+5\binom{10}{2}+9\binom{10}{3}+17\binom{10}{4}+33\binom{10}{5}...+1025\binom{10}{10}$$

3. A bug is crawling on a hexagon ABCDEF. It starts at point A, and picks goes clockwise with probability $\frac{1}{2}$ and counterclockwise with probability $\frac{1}{2}$. When it goes clockwise, it will move 1 point (i.e. from A to B). When it goes counterclockwise, it will move 2 points (i.e. from A to E). What is the probability that it will be at point A after 12 moves?

4. You have a sequence of n numbers, $\{a_1, a_2, ...a_n\}$. In one move, you can either: (a) add any real number k to the first $1 \geq j \geq n$ elements ($\{a_1, a_2, ...a_n\}$ becomes $\{a_1 + k, a_2 + k, ...a_j + k, a_{j+1}, ...a_n\}$) (b) replace the first $1 \geq j \geq n$ elements with their value mod any positive integer m. ($\{a_1, a_2, ...a_n\}$ becomes $\{a_1 \mod m, a_2 \mod m, ...a_j \mod m, a_{j+1}, ...a_n\}$) Prove you can always turn this into a strictly increasing sequence within $n + 1$ moves.

5. Given that for real x and y, $(x + \sqrt{1 + x^2})(y + \sqrt{1 + y^2}) = 1$, find $x + y$.