Monthly Contest 1
Due December 7, 2018

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problem's solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. **You may not view any website (including forums) unless otherwise stated in the problem.**

Problems

1. Alice and Bob play a card game. There are N cards in a pile, and they take turns taking 1, 2 or 3 card from the pile. Alice goes first, and whoever takes the last card wins. If Alice and Bob play perfectly (i.e. if they can win they will win), find all N where Alice wins.

2. Alice has written down a 3 digit base-7 number Bob looks at the number and mistakes it for a base-9 number. Carlos looks at the number and mistakes it for a base-5 number. When Alice's number is subtracted from Bob's number, we get twice Carlos' number. Find all possible numbers Alice could have written down, or prove that this is impossible.

3.

$$X = \frac{1}{2018}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2018}\right) \text{ and } Y = \frac{1}{2017}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2017}\right)$$

Which is bigger, X or Y?

4. 5 friends, A_1, A_2, A_3, A_4 and A_5 buy drinks from a vending machine. There are 20 kinds of drinks, and person A_i has i favorite drinks (not necessarily distinct from the other friends' favorites). However, some drinks are sold out. What is the least possible number of drinks sold out such that there is a $\frac{1}{2}$ chance that at least one person's favorite drinks are all sold out?

5. Triangle $\triangle ABC$ has $\overline{AB} = 13$, $\overline{BC} = 14$, and $\overline{AC} = 15$. Let O be the circumcenter of $\triangle ABC$, and P be the intersection of $\overline{AO}$ and the circumcircle of $\triangle ABC$ ($P \neq A$). Find $\overline{CP}$.