UNIT 1, ALGEBRA:

1. Determine all the integers n such that $n + [\sqrt{n}]$ is a perfect square.

2. Solve over the rational numbers the equation

$$x(x+1)(x+2) = 6.$$

Liviu Petre, in *Gazeta matematică*, 10/2010, E: 14088.

3. (a) Prove that for any integer k the equation $x^3 - 24x + k = 0$ has at most an integer solution.
(b) Show that the equation $x^3 + 24x - 2016 = 0$ has exactly one integer solution.

Cristian Alistar, at the second phase of the Romanian Olympiads, March 19, 2016.

UNIT 2, PLANE GEOMETRY:

4. Consider the right isosceles triangle ABC, with $A = 90^o$, and $M \in (BC)$ such that $m(\angle AMB) = 75^o$. On the interior angle bisector of $\angle MAC$ we take F such that $AB = BF$. Prove that
(a) $AM \perp BF$;
(b) ΔCFM is isosceles.

Adrian Bud, final phase of the Romanian Olympiads, April 20, 2016.

5. Given an angle $\angle AOB$ and P inside the angle, construct perpendiculars PA, PB, OC, and PD on OA, OB, AB, and AB, respectively. Prove that $AC = BD$.

From Robin Hartshorne, *Geometry: Euclid and Beyond,* Springer-Verlag, 2004.

6. Consider the rectangle $ABCD$ and points $M \in (AB)$, $N \in (BC)$, $P \in (CD)$, $Q \in (DA)$. Prove that $MP + NQ > AC$.

Gheorghe Stoica, in *Gazeta matematică*, S:11:325.

UNIT 3, SPACE GEOMETRY:

7. Show that in a regular quadrilateral pyramid two opposite lateral faces are perpendicular if and only if the angle between two adjacent lateral faces is 120^o.

Ion Tudor, second phase of the Romanian Olympiads, March 19, 2016.

8. (a) There exists a point equidistant from the four points that do not lie in the same plane.
(b) The planes perpendicular to the edges of a tetrahedron, passing through the midpoint of each edge, meet in a point.

9. Prove that the lines joining the midpoints of opposite edges in a tetrahedron meet in a point.

10. In a tetrahedron $ABCD$, let a, b, c, d be the lengths of heights from A, B, C, D. Let F be an arbitrary point inside $ABCD$. Let $\alpha, \beta, \gamma, \delta$ be the distances from F to faces BCD, CDA, DBA, and ABC, respectively. Then:

$$\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} + \frac{\delta}{d} = 1.$$

Problems **8, 9** and **10**, from Gheorghe Ţiţeica, *Problems in Geometry*, sixth edition, 1962.

Remarks, comments, complaints, and suggestions should be directed to Bogdan Suceavă, bsuceava@fullerton.edu