Monthly Contest 6
Due April 26, 2017

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. It would be preferred if you used 8.5 by 11 printer paper to write your solutions on and leave a small margin around the edges. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problems solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Leave answers in exact form unless otherwise specified. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems. You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. You may NOT view any book or website.

Problems

1. Let $\mathbb{N}$ be the set of positive integers. Find all strictly increasing functions f : $\mathbb{N} \to \mathbb{N}$ such that $f(2) = 2$ and $f(m)f(n) = f(mn)$ for all positive integers m and n.

2. 3 students each from Hilbert Middle School, Galois Middle School, and Noether Middle School sit in a row of 9 seats. In how many ways can we seat the students if no three students from the same school are seated next to each other?

3. Find the minimum term in the sequence
$\sqrt{\frac{7}{6}} + \sqrt{\frac{96}{7}}, \sqrt{\frac{8}{6}} + \sqrt{\frac{96}{8}}, \ldots, \sqrt{\frac{95}{6}} + \sqrt{\frac{96}{95}}$.

4. Prove that the sum of the squares of 3,4,5, or 6 consecutive integers is not a perfect square.

5. Find all the integer solutions of the system of equations:

$$
\begin{aligned}
ab + cd &= -1 \\
ac + bd &= -1 \\
ad + bc &= -1
\end{aligned}
$$