Monthly Contest 3
Due January 7, 2014

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problem's solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems.

You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. You may view any book or website (including forums) as long as you do not communicate the problem in any sort of manner (e.g. posting the problem statement or variations on a forum).

Problems

1. Find all reals k such that $2x^2 + 10x + y^2 + 4y = k$ has at least one solution.

2. Given $x + y = 1$ and $x^2 + y^2 = 2$, find $x^3 + y^3$.

3. Show that with 3 colors, it is impossible to color the plane such that no two points 1 unit apart are the same color.

4. Find all integers n such that $\dfrac{n^2 + 3}{2n + 4}$ is an integer as well.

5. Triangle ABC has side lengths a, b, and c such that the side with length a is opposite vertex A, the side with length b is opposite vertex B, and the side with length c is opposite vertex C. Point D is drawn on side BC such that AD bisects $\angle$ BAC. Point E is then drawn on BD such that AE bisects $\angle$ BAD. Find the area of triangle BAE in terms of a, b, and c.