Monthly Contest 2
Due December 3, 2014

Instructions

This contest consists of 5 problems, each worth 7 points. Problems may not be of equal difficulty. Please write your solution to every problem on a separate sheet of paper. If you use more than one sheet for a specific problem, please staple the sheets together. Do NOT staple together solutions to different problems. On the top of each sheet, include the contest number, problem number, and page number and the total number of pages of the problem's solution (not the total pages of all the solutions).

DO NOT put your name anywhere on your solutions. Doing so will result in instant disqualification. When you turn your solutions in, you will be asked to write a student code (which will be provided on the due date) on your solutions.

You must justify all answers to receive full credit. Answers without justification will receive few, if any, points. Partial credit will be given for significant progress into solving a problem. Do not feel discouraged if you cannot solve all the problems. It is already a great accomplishment if you are able to solve one of these problems.

You are NOT allowed to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, on forums, or any other means of communication. You may view any book or website (including forums) as long as you do not communicate the problem in any sort of manner (e.g. posting the problem statement or variations on a forum).

Problems

1. You have a bag with a fair coin and an unfair coin which has two heads. When you randomly pick one out of the bag and flip it twice, you get heads twice. What is the probability that you picked the unfair coin? Hint: Baye's Rule

2. Without using a calculator, find $\cos 36°$.

3. Prove that no infinite arithmetic sequence of natural numbers contains only prime numbers.

4. Consider the polynomial $x^n + ax^{n-1} + bx^{n-2} + \cdots$ with all real coefficients. Prove that if $(n-1)a^2 < 2b$ then the polynomial cannot have all real roots.

5. You have an n 1's on a chalkboard, and you can erase two numbers a and b on the board if you then write $(a+b)^2$ on the board. Let $f(n)$ be the minimum number remaining when you have managed to erase all the other numbers. Show that $f(n) \geq 4^{n-1}$ and find when equality occurs.

Additional Notes + Survey: Problem 1 was different from most contests because it was based on a topic (Bayesian Statistics) which isn't normally covered in a math competition. It would be helpful to know if you'd prefer more problems like this which cover topics in higher level mathematics or if you'd like to stick to traditional topics in algebra, geometry, number theory, and combinatorics. Feel free to add a small note with your thoughts, or email sjmathcircle@gmail.com with your responses.