

Introduction to Counting III

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Some hard problems?

1. What is the probability that a random 5-card poker hand has at least one card of each suit?
2. For “secret Santa”, 10 people each put their name in a collection and then randomly pull a name out. What’s the probability of a “successful” arrangement in which nobody gets their own name?

Warm-Ups

Different parts of a house, such as the exterior walls, doors, and window trim, can be painted different colors. So, for instance, a house might be all brown, or it might be red, white, and brown. In this problem, when we count the brown houses, we’d count both of those. That is, a house that has any parts painted in that color will be counted as having that color, so houses can be counted more than once.

In each of these problems, all the houses in the neighborhood are mentioned at least once in the question.

3. There are 20 red houses and 20 white houses. What is the largest number of houses this could be? What is the smallest?
4. There are 20 red houses, 20 white houses, and 5 red and white houses. How many houses are there in total?
5. There are r red houses, w white houses, and b houses that are both red and white. How many houses are there in total?
6. There are 20 red houses, 30 white houses, and 40 green houses. What is the largest number of houses this could be? What is the smallest?
7. There are 20 red houses, 30 white houses, 40 green houses, 10 red and white houses, 8 red and green houses, 6 white and green houses, and 2 red and white and green houses. How many houses are there in total?
8. Replace the numbers in the previous problem with variables (I suggest using multi-letter abbreviations, so r , w , g , rw , rg , wg , and rwg , just being careful not to confuse multi-letter names with multiplication). How many houses are there in total?

Traditional Exercises

9. How many integers in the range 1 through 1000 are divisible by at least one of 2, 3, or 5?
10. From AoPS forum: At Brokaw Hospital, six babies were born to six different women on Monday through Thursday of a particular week. Assuming that each baby was equally likely to be born on any of the four days, what is the probability that there was at least one baby born on each day?
11. How many positive integers are divisors of at least one of 10^{10} , 15^7 , and/or 18^{11} ?
12. How many equilateral triangles have at least two vertices among the 7 points formed by the vertices of a regular hexagon and its center?
13. How many of the first 100 positive integers can be expressed as a sum of three or fewer numbers selected from 1, 3, 9, 27, and 81? Each number may be used more than once.
14. How many 9-digit security codes (that may start with 0, unlike 9-digit numbers) contain the digits 1, 2, and 3 somewhere in them?