Mathematical Wrangle

The rules governing a mathematical wrangle closely follow those of a mathematical battle given in Appendix A of *Mathematical Circles (Russian Experience)* by Fomin, Gengkin and Itenberg. Adaptation provided by Sam Vandervelde.

Preliminaries

- A mathematical wrangle occurs between two groups of students, each with approximately the same number of members, typically three or four students per team.
- The jury composes a set of problems; about two more problems than the total number of participants. The solution to each problem will involve an explanation (not just a numerical answer) which requires a relatively short amount of time to present. Problems should span a range of topics and difficulty levels.
- Immediately prior to the wrangle each group is sequestered to work on the problem set as a team, for a duration based on the difficulty of the problems and the time available—perhaps an hour to ninety minutes.

Procedures

- A jury consisting of two or three judges keeps time, decides on scores, and generally presides over the wrangle. The decisions of the jury are final. (A jury may choose to entertain appeals if they wish.)
- Each team selects a captain, who serves as sole spokesperson for the team and participates in the captain's contest to determine order of play.
- The math wrangle commences with a short true/false question posed to the captains, who must solve the problem mentally, without the help of their team. The first person to raise their hand answers; if a correct answer is supplied then this captain decides whether to begin with the *right to challenge* or to pass this right to the other team. If the initial answer is incorrect then the other captain gets to make this decision.
- At each stage of the wrangle the team with the right to challenge chooses a problem from among those that have yet to be presented and challenges the opposing team to present a solution. In theory a team should choose a problem which they already understand, but a team may select a problem they have not solved. The captain announces a decision on behalf of the team.
- When challenged, the opposing team may choose to *accept* the challenge, in which case they present a solution. They may also opt to *return* the challenge, in which case the original team must attempt to present a solution.
- The team presenting a solution nominates one member who has not yet discussed a problem to provide an explanation. This person has up to five minutes to present as complete a solution to the problem as they are able. Time spent drawing diagrams or writing equations on the board is included in the five minutes. The presenter may briefly discuss the problem with their team prior to stepping to the board, but may not consult with their team while describing their solution.
- The other team then nominates one member who has not yet given a rebuttal to respond to the solution just presented. This person has up to three minutes to point out any flaws or omissions in the explanation or to illustrate how that explanation might be shortened or made more elegant. *The rebuttal must address the solution presented.* In particular, an alternate solution should not be given at this point. The responder can discuss their rebuttal briefly with their team, but then may not consult with their team while speaking.
- Once both teams have discussed the chosen problem the jury, at its discretion, may direct questions towards either the original presenter or the responder. They may also supply further commentary upon the problem or its solution, if desired.
- When all discussion of the problem has come to a close, the jury distributes the points available among the three parties involved: the original team, the responding team, and themselves (the jury). For instance, if the response indicates how to substantially improve

a complete proof, the jury may award several points to the responding team instead of to the original team. If no team makes significant headway towards a solution, then the jury might award the majority of the available points to themselves.

- With one exception the right to challenge then passes to the other team, and the wrangle enters its next stage. The exception occurs when a team returns a challenge and the original team is unable to make significant progress towards a solution (defined as receiving 3 or fewer points), in which case the right to challenge remains with the original team. The wrangle continues until the allotted number of solutions have been attempted. It is not necessary (or desirable) for every problem to be discussed.

Etiquette

- Team members should listen attentively or consult with one another quietly as solutions or responses are being presented. Heckling is frowned upon.

- Team members responding to a solution should always speak respectfully of the work done by the opposing team. For instance, one might refer to a "trifling oversight" rather than a "stupid mistake" In the same spirit, a response might begin "The opposing team has made significant progress towards a solution to this problem. I would now like to propose a nice approach that circumvents the algebraic difficulties they encountered."

- All students should honor the decisions made by the judges. The judges, for their part, will act impartially to the best of their ability, and encourage all the team members in their efforts. If the jury is willing to entertain appeals, the team captain may appeal a decision immediately following the awarding of points for a particular problem. The captain might approach the jury and explain, "We felt that the points should have been allocated as follows, for these reasons ...." The jury will then announce a change of score, if they have been swayed by the appeal. A team may only appeal one scoring decision per wrangle.

Scoring and Strategy

- It is generally helpful for team members to understand solutions to multiple problems. Teams might also choose to appoint an "expert" for each problem.

- Each problem is initially worth 7 points. Should a team accept a challenge to solve a problem, then the jury will distribute these 7 points at the conclusion of its discussion. However, should a team return a challenge, then the problem increases in value to 10 points. In this case the jury will distribute 10 points among the teams and themselves, rather than 7.

- Recall that if a team makes a challenge, that challenge is returned, and it comes to light that the original team is unable to make significant progress towards a solution (defined as receiving 3 or fewer of the available 10 points), then the right to challenge does *not* pass to the other team.

- Therefore a team that is challenged to solve a particular problem is faced with a dilemma if they do not have a solution. If they accept the challenge anyway, then they lose the opportunity to score a full 7 points, although they won't necessarily fall behind if they have at least a partial solution. However, if they return the challenge and the original team has a proof, then they might fall behind by up to 10 points. On the other hand, if the first team was bluffing, then the right to challenge remains with the original team, so they will immediately have another opportunity to present a solution.

Conclusion

- Once the predetermined number of problems have been presented the mathematical wrangle comes to a close. The jury announces the final scores and declares a victor.

- Prizes (when provided) are distributed to all participants on both teams equally. It is also recommended that everyone celebrate together afterwards, preferably with good food and beverages.