Mathematical Wrangle

Practice Problem Set

1. Is it possible to draw ten lines in the plane which intersect in precisely thirty-five points?

2. Let a, b, c, d be distinct positive integers such that $ab = cd$. Prove that the sum $a^n + b^n + c^n + d^n$ is composite for all positive integers n.

3. The lengths of the sides of triangle ABC measure $AB = 5$, $AC = 4$, and $BC = 3$. Let M be the midpoint of side AC. Find, with proof, the length of the shortest path that begins at M, reaches side AB, heads over to side BC, then returns to its starting point.

4. A father decides to split 20 gold coins (of equal value) among his five sons. He declares that the oldest son will propose a distribution of coins, at which point the second son (after hearing the first plan) will have the opportunity to propose a different distribution. All five sons will then vote for one of the two proposals; the plan receiving the majority of votes will be implemented. Assume that all sons will all act in a way to maximize their earnings but will vote for the older son's plan, all else being equal. Determine what will transpire.