ANALYTIC GEOMETRY

1. *(1953 AMC 12, ♯49)* The coordinates of A, B, and C are $(5,5)$, $(2,1)$, and $(0,k)$ respectively. The value of k that makes $AC+BC$ as small as possible is:

   (A) 3 (B) $4\frac{1}{2}$ (C) $3\frac{6}{7}$ (D) $4\frac{5}{6}$ (E) $2\frac{1}{7}$

2. *(1958 AMC 12, ♯27)* The points $(2,-3)$, $(4,3)$, and $(5,k/2)$ are on the same straight line. The value(s) of k is (are):

   (A) 12 (B) -12 (C) ± 12 (D) 12 or 6 (E) 6 or $6\frac{2}{3}$

3. *(1958 AMC 12, ♯35)* A triangle is formed by joining three points whose coordinates are integers. Then the area of the triangle:

   (A) must be an integer (B) may be irrational (C) must be irrational (D) must be rational (E) will be an integer only if the triangle is equilateral

4. *(Mathematics Student Journal)* Prove that if the vertices of a triangle have coordinates that are integers, then the triangle cannot be equilateral.

5. *(1957 AMC 12, ♯25)* The vertices of triangle PQR have coordinates as follows: $P(0, a)$, $Q(b, 0)$, $R(c, d)$, where a, b, c, and d are positive. The origin and point R lie on opposite sides of PQ. The area of triangle PQR may be found from the expression:

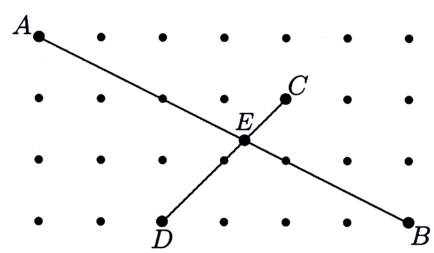
(A) $\dfrac{ab + ac + bc + cd}{2}$ (B) $\dfrac{ac + bd - ab}{2}$ (C) $\dfrac{ab - ac - bd}{2}$ (D) $\dfrac{ac + bd + ab}{2}$
(E) $\dfrac{ac + bd - ab - cd}{2}$

6. *(1967 AMC 12, ♯40)* Located inside equilateral triangle ABC is a point P such that $PA = 6$, $PB = 8$, and $PC = 10$. To the nearest integer, the area of the triangle ABC is:

(A) 159 (B) 131 (C) 95 (D) 79 (E) 50

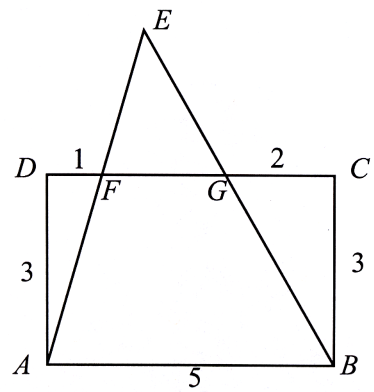
7. Let $ABCD$ be a rectangle with $AB = 8$ and $AD = 12$. Points X and Y are on $\overline{AB}$ and $\overline{CD}$, respectively, such that $AX = 7/3$ and $DY = 1$. $\overline{XY}$ meets diagonal $\overline{AC}$ at point P. Find BP.

8. *(2000 AMC 10, ♯16)* The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segment AB meets segment CD at E. Find the length of the segment AE.



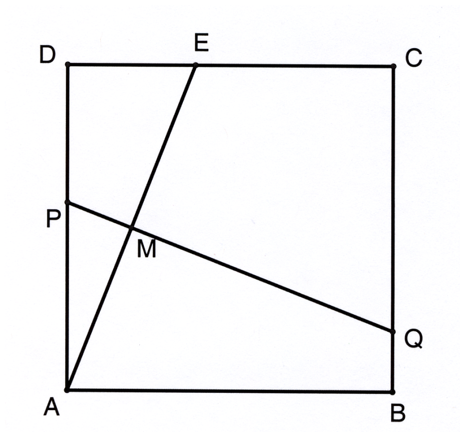
(A) $4\sqrt{5}/3$ (B) $5\sqrt{5}/3$ (C) $12\sqrt{5}/7$ (D) $2\sqrt{5}$ (E) $5\sqrt{65}/9$

9. *(2003 AMC 10B, ♯20 and 2003 AMC 12B, ♯14)* In rectangle $ABCD$, we have $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ with $DF = 1$ and $GC = 2$, and lines AF and BG intersect at E. What is the area of $\triangle AEB$?



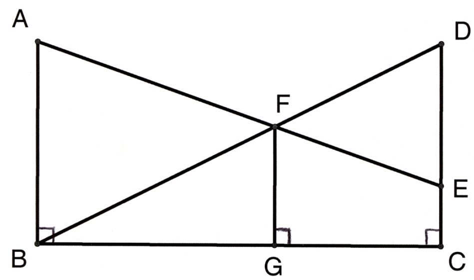
(A) 10 (B) $\frac{21}{2}$ (C) 12 (D) $\frac{25}{2}$ (E) 15

10. *(1972 AMC 12, ♯13)* Inside square $ABCD$ with sides of length 12 inches, segment AE is drawn, where E is the point on DC which is 5 inches from D. The perpendicular bisector of AE is drawn and intersects AE, AD, and BC at points M, P, and Q, respectively. Then the ratio of the segment PM to MQ is:



(A) 5:12 (B) 5:13 (C) 5:19 (D) 1:4 (E) 5:21

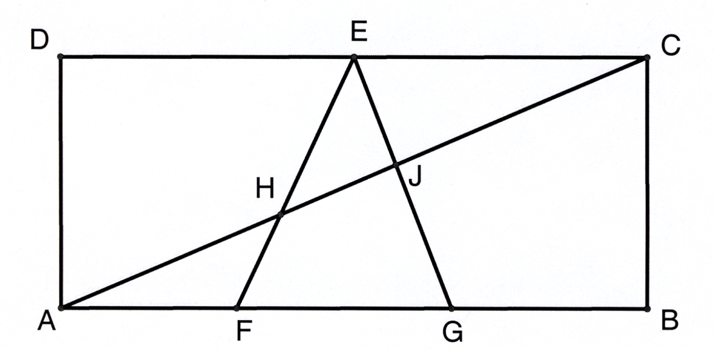
11. In the diagram below, we have $DE = 2EC$ and $AB = DC = 20$. Find the length of FG.



12. *(2001 AMC 12, ♯20)* Points $A = (3, 9)$, $B = (1, 1)$, $C = (5, 3)$, and $D = (a, b)$ lie in the first quadrant and are the vertices of quadrilateral $ABCD$. The quadrilateral formed by joining the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ is a square. What is the sum of the coordinates of point D?

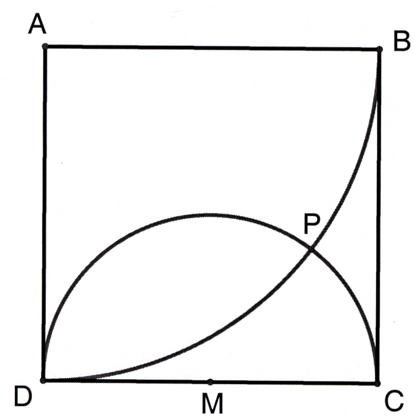
(A) 7 (B) 9 (C) 10 (D) 12 (E) 16

13. *(2001 AMC 12, ♯22)* In rectangle $ABCD$, points F and G lie on $\overline{AB}$ so that $AF = FG = GB$ and E is the midpoint of $\overline{DC}$. Also, $\overline{AC}$ intersects $\overline{EF}$ at H and $\overline{EG}$ at J. The area of rectangle $ABCD$ is 70. Find the area of triangle EHJ.



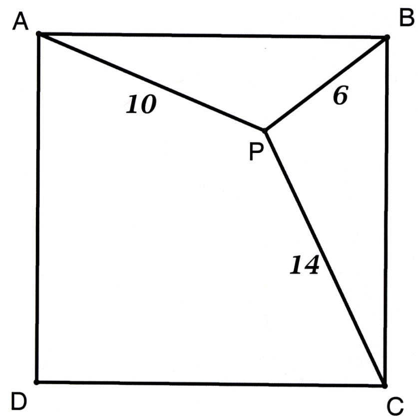
(A) $\frac{5}{2}$ (B) $\frac{35}{12}$ (C) 3 (D) $\frac{7}{2}$ (E) $\frac{35}{8}$

14. *(2003 AMC 12, ♯17)* Square $ABCD$ has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects a circle with radius 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?



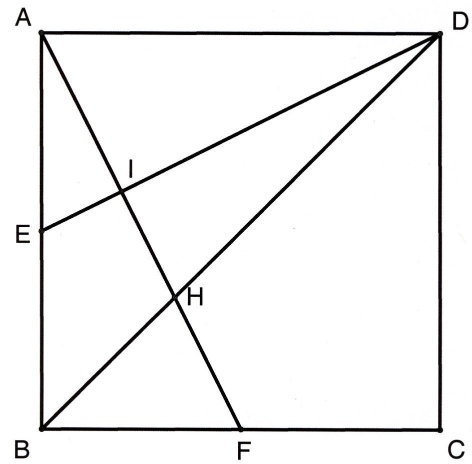
(A) 3 (B) $\frac{16}{5}$ (C) $\frac{13}{4}$ (D) $2\sqrt{3}$ (E) $\frac{7}{2}$

15. In the figure shown, point P is located inside square $ABCD$. If $PA = 1$, $PB = 6$ and $PC = 14$, find the area of the square.



(A) $8\sqrt{58}$ (B) 140 (C) 232 (D) $12\sqrt{58}$ (E) 464

16. If $ABCD$ is a 2×2 square , E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, $\overline{AF}$ and $\overline{DE}$ intersect at I, and $\overline{BD}$ and $\overline{AF}$ intersect at H, then the area of the quadrilateral $BEIH$ is:



(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{7}{15}$ (D) $\frac{8}{15}$ (E) $\frac{3}{5}$

17. Suppose that a line ℓ divides a rectangle $ABCD$ into two pieces with equal area. Prove that ℓ passes through the intersection of the diagonals of $ABCD$.

18. *(1993 AIME, $\sharp$13)* Jenny and Kenny are walking in the same direction, Kenny at 3 feet per second and Jenny at 1 foot per second, on parallel paths that are 200 feet apart. A tall circular building 100 feet in diameter is centered midway between the paths. At the instant when the building first blocks the line of sight between Jenny and Kenny, they are 200 feet apart. Let t be the amount of time, in seconds, before Jenny and Kenny can see each other again. If t is written as a fraction in lowest terms, what is the sum of the numerator and the denominator?

19. *(1942 Eötvös Competition, Hungary)* Let A', B', and C' be points on the sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively, of an equilateral triangle ABC such that $AC' = 2C'B$, $BA' = 2A'C$, and $CB' = 2B'A$. Prove that the lines AA', BB', and CC' enclose a triangle whose area is $\frac{1}{7}$ that of ABC.

20. *(Cristian S. Calude Mathematics Competition, Team Contest, Grades 11-12, 2008, Romania)* In $\triangle ABC$, M is on $\overline{BC}$, N is on $\overline{AB}$ and P is on $\overline{AC}$ so that $\frac{NB}{NA} = \frac{1}{2}$, $\frac{PC}{PA} = \frac{1}{3}$, $\frac{BM}{MC} = \frac{2}{3}$. Let U be the intersection of AM and NP. Find $\frac{AU}{UM}$.

21. *(The Yearly Competition of Gazeta Matematica, 1991, 9th and 10th grade)* Let M, N, and P be the midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ of the triangle ABC, respectively. On the perpendicular bisectors of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ choose, inside the triangle, points A', B', and C' such that $\dfrac{MC'}{AB} = \dfrac{NA'}{BC} = \dfrac{PB'}{AC}$. Show that the triangles ABC and $A'B'C'$ have the same centroid.

22. *(2003 Romanian Mathematics Olympiad, Final Round, 11th grade)* Find the locus of all points M in the plane of a rhombus $ABCD$ for which the following holds:

$$MA \cdot MC + MB \cdot MD = AB^2$$