San Jose Math Circle

## DISTANCE, TIME, AND RATE

1. (1950 AMC 12,  $\sharp$ 27) A car travels 120 miles from A to B at 30 miles per hour but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to:

(A) 33 mph (B) 34 mph (C) 35 mph (D) 36 mph (E) 37 mph

2.  $(1952 AMC 12, \sharp 10)$  An automobile went up a hill at a speed of 10 miles per hour and down the same distance at a speed of 20 miles per hour. The average speed for the round trip was:

(A)  $12\frac{1}{2}$  mph (B)  $13\frac{1}{3}$  mph (C)  $14\frac{1}{2}$  mph (D) 15 mph (E) none of these

3. (2000 State Sprint \$26) Janelle averages 40 kilometers per hour biking on level ground. She averages 60% of her level-ground speed riding uphill, and she averages 120% of her level-ground speed riding downhill. The course is level for 5 kilometers, uphill for 6 kilometers, and then downhill for 6 kilometers. How many kilometers per hour is her average speed for the entire course?

4.  $(1958 \ AMC \ 12, \ \sharp 24)$  A man travels *m* feet due north at 2 minutes per mile. He returns due south to his starting point at 2 miles per minute. The average rate in miles per hour for the entire trip is:

(A) 75 (B) 48 (C) 45 (D) 24

(E) impossible to determine without knowing the value of m

5. (1952 AMC 12,  $\sharp 32$ ) K travels 30 minutes less time than M to travel a distance of 30 miles. K travels  $\frac{1}{3}$  mile per hour faster than M. If x is K's rate of speed in miles per hour, then K's time for the distance is:

(A) 
$$\frac{x + (1/3)}{30}$$
 (B)  $\frac{x - (1/3)}{30}$  (C)  $\frac{30}{x + (1/3)}$  (D)  $\frac{30}{x}$  (E)  $\frac{x}{30}$ 

6.  $(1952 \ AMC \ 12, \ \sharp 48)$  Two cyclists, k miles apart, and starting at the same time, would be together in r hours if they traveled in the same direction, but would pass each other in t hours if they traveled in opposite directions. The ratio of the speed of the faster cyclist to that of the slower is:

(A) 
$$\frac{r+t}{r-t}$$
 (B)  $\frac{r}{r-t}$  (C)  $\frac{r+t}{r}$  (D)  $\frac{r}{t}$  (E)  $\frac{r+k}{t-k}$ 

7. (1952 AMC 12, \$\$\\$25) A powderman set a fuse for a blast to take place in 30 seconds. He ran away at a rate of 8 yards per second. Sound travels at the rate of 1080 feet per second. When the powderman heard the blast, he had run approximately:

(A) 200 yd. (B) 352 yd. (C) 300 yd. (D) 245 yd. (E) 512 yd.

8. (1953 AMC 12, \$31) The rails on a railroad are 30 feet long. As the train passes over the point where the rails are joined, there is an audible click. The speed of the train in miles per hour is approximately the number of clicks heard in:

(A) 20 seconds (B) 2 minutes (C)  $1\frac{1}{2}$  minutes (D) 5 minutes

(E) none of these

9. (1950 AMC 12, #28) Two boys A and B start at the same time to ride from Port Jervis to Poughkeepsie, 60 miles away. A travels 4 miles per hour slower than B. B reaches Poughkeepsie and at once turns back meeting A 12 miles from Poughkeepsie. The rate of A was:

(A) 4 mph (B) 8 mph (C) 12 mph (D) 16 mph (E) 20 mph

10. (1950 AMC 12, \$\$50) A privateer discovers a merchantman 10 miles to leeward at 11:45 a.m. and with a good breeze bears down upon her at 11 mph, while the merchantman can only make 8 mph in her attempt to escape. After a two hour chase, the top sail of the privateer is carried away; she can now make only 17 miles while the merchantman makes 15. The privateer will overtake the merchantman at:

(A) 3:45 p.m. (B) 3:30 p.m. (C) 5:00 p.m. (D) 2:45 p.m. (E) 5:30 p.m.

11. (1954 AMC 12,  $\sharp 48$ ) A train, an hour after starting, meets with an accident which detains it a half hour, after which it proceeds at  $\frac{3}{4}$  of its former rate and arrives  $3\frac{1}{2}$  hours late. Had the accident happened 90 miles further along the line, it would have arrived only 3 hours late. The length of the trip in miles was:

(A) 400 (B) 465 (C) 600 (D) 640 (E) 550

12.  $(1955 \ AMC \ 12, \sharp 41)$  A train traveling from Aytown to Beetown meets with an accident after 1 hr. It is stopped for  $\frac{1}{2}$  hours, after which it proceeds at four-fifths of its usual rate, arriving at Beetown 2 hr late. If the train had covered 80 miles more before the accident, it would have been just 1 hr late. The usual rate of the train is:

(A) 20 mph (B) 30 mph (C) 40 mph (D) 50 mph (E) 60 mph

- 13. (1956 AMC 12, \$\\$32) George and Henry started a race from opposite ends of the pool. After a minute and a half, they passed each other in the center of the pool. If they lost no time in turning and maintained their respective speeds, how many minutes after starting did they pass each other the second time?
  - (A) 3 (B)  $4\frac{1}{2}$  (C) 6 (D)  $7\frac{1}{2}$  (E) 9

14. (1960 AMC 12, \$\$34) Two swimmers, at opposite ends of a 90-feet pool, start to swim the length of the pool, one at a rate of 3 feet per second, the other at 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

(A) 24 (B) 21 (C) 20 (D) 19 (E) 18

- 15. (1955 AMC 12, \$\$50) In order to pass B going 40 mph on a two-lane highway A, going 50 mph, must gain 30 feet. Meantime, C, 210 feet from A, is headed toward him at 50 mph. If B and C maintain their speeds, then, in order to pass safely, A must increase his speed by:
  - (A) 30 mph (B) 10 mph (C) 5mph (D) 15 mph (E) 3 mph



This figure is not drawn to scale

16. (1991 AMC 12, \$\pm 11) Jack and Jill run 10 kilometers. They start at the same point, run 5 kilometers up a hill, and return to the starting point by the same route. Jack has a 10-minute head start and runs at a rate of 15 km/hr uphill and 20 km/hr downhill. Jill runs 16 km/hr uphill and 22 km/hr downhill. How far from the top of the hill are they when they pass going in opposite directions?

(A) 
$$\frac{5}{4}$$
 km (B)  $\frac{35}{27}$  km (C)  $\frac{27}{20}$  km (D)  $\frac{7}{3}$  km (E)  $\frac{28}{9}$  km

- 17. (1951 AMC 12, \$50) Tom, Dick, and Harry started out on a 100-mile journey. Tom and Harry went by automobile at the rate of 25 mph, while Dick walked at the rate of 5 mph. After a certain distance, Harry got off and walked on at 5 mph, while Tom went back for Dick and got him to the destination at the same time that Harry arrived. The number of hours required for the trip was:
  - (A) 5 (B) 6 (C) 7 (D) 8 (E) none of these answers

18. (2005 Harker Math Invitational, Team #6) Train A is 200 ft long. Train B is 400 ft long. They run on parallel tracks each traveling at constant speeds. When moving in the same direction, Train A passes Train B in 15 seconds. In the opposite direction, they pass each other in 5 seconds. How fast is each train traveling? Express your anser in ft/sec.

19. (2009 AMC 10A, *‡20*) Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

(A) 20 (B) 30 (C) 55 (D) 65 (E) 80

20. (2000 State Team #1) A train traveling at 60 miles per hour reaches a tunnel. The front of the train enters the tunnel, and 40 seconds later the front of the engine exits the tunnel. How many feet are in the length of the tunnel?

21. (1991 National Sprint  $\sharp 28$ ) A man is running through a train tunnel. When he is  $\frac{2}{5}$  of the way through, he hears a train that is approaching the tunnel from behind him at a speed of 60 mph. Whether he runs ahead or runs back, he will reach an end of the tunnel at the same time the train reaches that end. At what rate, in mph, is he running? (Assume that he runs at a constant rate.)

22. (2004 Harker Math Invitational, Team \$\$) A battalion 20 miles long advances 20 miles. During this time, a messenger on a horse travels from the rear of the battalion to the front and immediately turns around, ending up precisely at the rear of the battalion upon the completion of the 20-mile journey. How far has the messenger traveled?

23. (2007 Chapter Target #8) Dr. Lease leaves his house at exactly 7:20 a.m. every morning. When he averages 45 miles per hour, he arrives at his workplace five minutes late. When he averages 63 miles per hour, he arrives five minutes early. What speed should Dr. Lease average to arrive at his workplace precisely on time? Express your answer as a decimal to the nearest tenth.

24. (2004 Harker Math Invitational, Individual \$20) Ron has an appointment with George to meet at a certain place. If Ron walks 4 miles per hour, he would be late and George would have to wait for him 10 minutes, but if Ron increased his speed by 25%, then he would have to wait for George for 20 minutes. Find the distance Ron has to cover.

25. (2007 State Sprint #10) Louisa ran at an average speed of 5 miles per hour along an entire circular path. Calvin ran along the same path in the opposite direction at an average speed of six miles per hour. It took Calvin 30 minutes less than it took Louisa to run the full path once. How many miles did Louisa run when she completed one circular path?

26. (1959 AMC 12,  $\sharp$ 30) A can run around a circular track in 40 seconds. B, running in the opposite direction, meets A every 15 seconds. What is B's time to run around the track, expressed in seconds?

(A) 
$$12\frac{1}{2}$$
 (B) 24 (C) 25 (D)  $27\frac{1}{2}$  (E) 55

27. (1973 AMC 12,  $\sharp$ 29) Two boys start moving from the same point A on a circular track but in opposite directions. Their speeds are 5 ft per sec and 9 ft per sec. If they start at the same time and finish when they first meet at the point A again, then the number of times they meet, excluding the start and the finish, is:

(A) 13 (B) 25 (C) 44 (D) infinity (E) none of these

28. (1973 AMC 12, #34) A plane flew straight against a wind between two towns in 84 minutes and returned with that wind in 9 minutes less than it would take in still air. The number of minutes for the return trip was:

(A) 54 or 18 (B) 60 or 15 (C) 63 or 12 (D) 72 or 36 (E) 75 or 20

29. (2000 State Sprint  $\sharp$ 29) Janene and Emily plan to go on a marathon training run. Emily arrives late, so Janene starts running 16 minutes before Emily. Janene runs at an average rate of 9 minutes per mile, and Emily runs at an average rate of  $8\frac{1}{4}$  minutes per mile. Assuming that both girls started at the same location and ran the same route, how many minutes will Emily take to catch up to Janene?

30. (2008 AIME II, #2) Rudolph bikes at a constant rate and stops for a five-minute break at the end of every mile. Jennifer bikes at a constant rate which is three-quarters the rate that Rudolph bikes, but Jennifer takes a five-minute break at the end of every two miles. Jennifer and Rudolph begin biking at the same time and arrive at the 50-mile mark at exactly the same time. How many minutes has it taken them?

31. (2007 AIME I,  $\sharp 2$ ) A 100 foot long moving walkway moves at a constant rate of 6 feet per second. Al steps onto the start of the walkway and stands. Bob steps onto the start of the walkway two seconds later and strolls forward along the walkway at a constant rate of 4 feet per second. Two seconds after that, Cy reaches the start of the walkway and walks briskly forward beside the walkway at a constant rate of 8 feet per second. At a certain time, one of these three persons is exactly halfway between the other two. At that time, find the distance in feet between the start of the walkway and the middle person.

32. (1987 AIME, \$10) Al walks down to the bottom of an escalator that is moving up and he counts 150 steps. His friend, Bob, walks up to the top of the escalator and counts 75 steps. If Al's speed of walking (in steps per unit time) is three times Bob's speed, how many steps are visible on the escalator at any given time? (Assume that this number is constant).

33.  $(1993 \ AIME, \sharp 13)$  Jenny and Kenny are walking in the same direction, Kenny at 3 feet per second and Jenny at 1 foot per second, on parallel paths that are 200 feet apart. A tall circular building 100 feet in diameter is centered midway between the paths. At the instant when the building first blocks the line of sight between Jenny and Kenny, they are 200 feet apart. Let t be the amount of time, in seconds, before Jenny and Kenny can see each other again. If t is written as a fraction in lowest terms, what is the sum of the numerator and the denominator?

34. (1961 AMC 12,  $\ddagger$  37) In racing over a distance d at uniform speed, A can beat B by 20 yards, B can beat C by 10 yards, and A can beat C by 28 yards. Then d, in yards, equals:

(A) not determined by the given information (B) 58 (C) 100 (D) 116

(E) 120

35. (2008 AIME I,  $\sharp 3$ ) Ed and Sue bike at equal and constant rates. Similarly, they jog at equal and constant rates, and they swim at equal and constant rates. Ed covers 74 kilometers after biking for 2 hours, jogging for 3 hours, and swimming for 4 hours, while Sue covers 91 kilometers after jogging for 2 hours, swimming for 3 hours, and biking for 4 hours. Their biking, jogging, and swimming rates are all whole numbers of kilometers per hour. Find the sum of the squares of Ed's biking, jogging, and swimming rates.

36. (2008 AIME I,  $\sharp 12$ ) On a long straight stretch of one-way single-lane highway, cars all travel at the same speed and all obey the safety rule: the distance from the back of the car ahead to the front of the car behind is exactly one car length for each 15 kilometers per hour of speed or fraction thereof (Thus the front of a car traveling 52 kilometers per hour will be four car lengths behind the back of the car in front of it.) A photoelectric eye by the side of the road counts the number of cars that pass in one hour. Assuming that each car is 4 meters long and that the cars can travel at any speed, let M be the maximum whole number of cars that can pass the photoelectric eye in one hour. Find the quotient when M is divided by 10.