San Jose Math Circle January 31, 2009

VIETE'S EQUATIONS - PART II

1. *(1986 AIME, ♯1)* What is the sum of the solutions of the equation: $\sqrt[4]{x} = \dfrac{12}{7 - \sqrt[4]{x}}$?

2. *(1983 AIME, ♯3)* What is the product of the real roots of the equation

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}\,?$$

3. *(1975 AMC 12, ♯22)* If p and q are primes and $x^2 - px + q = 0$ has distinct positive integral roots, then which of the following statements are true?

(I) The difference of the roots is odd.

(II) At least one root is prime.

(III) $p^2 - q$ is prime.

(IV) $p + q$ is prime.

(A) I only (B) II only (C) II and III only (D) I, II, and IV only

(E) All are true

4. *(1975 AMC 12, ♯22)* If p, q, and r are distinct roots of $x^3 - x^2 + x - 2 = 0$, then $p^3 + q^3 + r^3$ equals:

(A) -1 (B) 1 (C) 3 (D) 5 (E) none of these

5. If r_1, r_2, $\ldots$, r_{1000} are the roots of $x^{1000} - 10x + 10 = 0$, find $r_1^{1000} + r_2^{1000} + \cdots + r_{1000}^{1000}$.

6. *(1976 AMC 12, ♯30)* How many distinct ordered triples (x, y, z) satisfy the equations:

$$\begin{cases} x + 2y + 4z & = 12 \\ xy + 4yz + 2xz & = 22 \\ xyz & = 6 \end{cases}$$

(A) none (B) 1 (C) 2 (D) 4 (E) 6

7. Solve the following system of equations:

$$\begin{cases} x + y + z &= 2 \\ x^4 + y^4 + z^4 &= 8 \\ x^5 + y^5 + z^5 &= 32 \end{cases}$$